Response to "Review of the J-value literature –Final Report [to] The Health and Safety Executive" by Michael Spackman, NERA Economic Consulting
www.hse.gov.uk/nuclear/j-value-report.htm

Philip Thomas
Roger Jones

Risk Management, Reliability and Maintenance Group
School of Engineering and Mathematical Sciences
City University
Northampton Square
London EC1V 0HB

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Summary

The J-value framework (J for Judgement) is an integrated methodology that generates objective advice on how much should be spent to avert human harm and environmental loss. This article is a response to a literature review of the J-value by Michael Spackman of NERA Economic Consulting recently published on the HSE website.

While Mr Spackman has been fulsome in his praise of some of the J-value articles he has examined, he has omitted from consideration all of the J-value papers concerned with valuing human harm from nuclear radiation doses. As a result, he overlooks the problem of how to value correctly harmful effects that may be delayed by decades. This issue is of vital importance to worker and public protection in the context of not only of the Royal Navy's nuclear submarine reactors but also the civil nuclear power programme, the expansion of which is being relied upon by the Government to maintain the UK's electricity supplies without adding to global warming. See Section 2.

The review seems to display a view of utility that is inconsistent with mainstream thought. Mr Spackman goes against established economic, actuarial and political precedent when he seeks to restrict the region of applicability of utility functions to individuals only. He seems unaware of the Cobb-Douglas utility function that is in frequent use by economists. The use of a consistent unit of currency, founded in people's economic experience, means that his remarks on the unit of currency (or *numéraire*) have no force. His comments on the scaling factor to be applied to a utility difference are mathematically incorrect. Section 3 deals with these aspects.

Concerning the value of risk-aversion, it is clear that Mr Spackman has preserved his intellectual allegiance to the figure of 1.5 once advocated by the Government. However, Government advice was updated about 10 years ago, and the central recommendation is now 1.0. The J-value results, giving risk-aversion as between 0.82 and 0.85, are close to an independently derived economic result of 0.83 and within 20% of the Government's central view. Unlike Mr Spackman's preferred value, the risk-aversions coming from the J-value are in line with Treasury guidance that the

figure is most likely to be just above or just below unity. Section 4 discusses this issue.

A clear inconsistency arises between Mr Spackman's recent recommendation to HSE of the NERA valuation model, based on putting a value on life expectancy, and his lack of enthusiasm for the J-value model *because* it values life expectancy, as embodied by his remark that "as a principle this is very questionable". Only recently a NERA Project Team, of which Mr Spackman was the in-house member, commended to HSE a model that treated "the VPF as reflecting literally the 'value of remaining life', or, more specifically, the sum of the value of increments in life expectancy, where the increments in total add up to the life expectancy itself". This NERA model was put forward as "a mathematical structure, which is an advance on any other of which we are aware, and which we recommend as suitable, in the present state of knowledge, for practical application". See Section 5.

The review's charges against the J-value of lack of transparency and even of obscurity are without foundation. A distinguishing feature of the J-value framework is that the assumptions have been laid out clearly in the large number of peer-reviewed, supporting papers. Nothing is implicit in the J-value approach – the assumptions are clear and all the data are explicit and testable economic and actuarial figures. See Section 6.

Overall, as discussed in Section 7, the review appears to display a bias against innovation. Mr Spackman seems to believe that the theoretical *status quo* in health and safety is either entirely adequate, or at least cannot be improved. Thus, while it is useful in stimulating debate on the new J-value method, it is not clear what function Mr Spackman's review can serve beyond this.

Keywords: J-value, utility, risk aversion, value of a life year, transparency, innovation

1. Introduction

The J-value framework (J for Judgement) comprises an integrated set of methods that provide, for the first time, fully objective advice on how much should be spent to avert human harm and environmental loss. Michael Spackman of NERA Economic Consulting, acting as a one-person "Project Team", has carried out a literature review of the J-value method in the context of the MoD's desire to maintain and improve the safety of its submarine nuclear reactors. This has been published recently on the HSE's website (Spackman, 2011).

Mr Spackman has been fulsome in his praise of some of the articles he has examined: "These are useful and impressive papers" which are "superbly clear and a fine read" and illustrate "how the valuation of fatality risks varies across different regulatory regimes, and varies even more in potential and actual application". Furthermore, Mr Spackman endorses strongly the need for interdisciplinary work in the field of health and safety:

> "It is good to see interdisciplinary work and new ideas in any field and in particular in applications such as analysis of the costs and benefits of potential spending to

> improve health and safety. There are important issues in the field that remain persistently resistant to formal analysis and no one discipline alone is equipped to handle them.
>
> In the case of the J-value literature the authors have shown exceptional energy and enthusiasm and an interest in and willingness to delve into the economics literature. Their publication in a technological journal of comparisons of regulatory conventions and applications was a notable and valuable achievement."

Moreover he recognises the inconsistencies associated with the application of the currently used value of a [temporarily] prevented fatality (VPF) approach:

> "the J-value package may have made the presentation especially effective in the chosen journal, to a wide audience not previously familiar with the VPF concept and the apparent inconsistencies in its application."

for, according to Mr Spackman, although safety professionals have long felt that such discrepancies abounded in practice, they have not been able to communicate their unease either to those taking decisions or to the general public. Hence he writes:

> "The checks against practical applications tell a vivid story, albeit one largely familiar to those working in general safety regulation. In terms solely of expected lives saved the TPWS [*Train Protection and Warning System*] gave very poor value for money (VFM) even with the LQI [*Life Quality Index*] -based VPF [*Mr Spackman's description of the J-value*]. The ERTMS [*European Rail Transport Management System*] would have given appalling VFM (as was recognised by HSC [*Health and Safety Commission*] at the time that it recommended against it). The drugs, on an LQI basis, generally gave very good VFM. Early measures against BSE [*Bovine Spongiform Encephalopathy*] gave very good VFM and countermeasures after March 1996 very bad VFM. The BNFL Tc-99 plants gave appalling VFM, but this policy was implemented; the Krypton options gave worse VFM but were not implemented. The petrol station VOC [*Volatile Organic Compounds*] measures gave VFM that was poor even with the LQI-based VPF, but only by a factor of about two.
>
> The discussion in Thomas et al (2006.2) suggests that it is “difficult to understand the logic” of disparities as severe as the spending of huge sums on the BNFL Technetium plant while not approving a drug that would save many more life-years. It suggests as noted earlier that the J-value has an “ability to translate a variety of cost-benefit formats into a common yardstick” and that “its adoption could lead to better targeting of health and safety expenditure in all areas of the economy”."

On the other hand, there are a number of problems with Mr Spackman's review. The first and most obvious one is that, although it was commissioned in the context of nuclear safety, the review omitted from its consideration every J-value paper related to the risks from nuclear radiation doses. This is a strange omission. About half the J-value papers published in *Process Safety and Environmental Protection,* as measured by page-count, were left out of Mr Spackman's literature review. Inevitably, this omission renders the review partial and incomplete. The 8 nuclear J-value papers published in *Process Safety and Environmental Protection* but omitted from consideration are listed in Section 2.

The review's decision not to take account of specifically nuclear hazards is a fundamental deficiency, because most radiation doses, whether as a result of an accident or from a continuing exposure, lead to health effects that, should they occur, will be delayed by decades. If the radiation dose is below the high level of about 500 mSv, there will be no immediate effects, and any mortality, should it happen, will be delayed by between 10 and 40 years or more. Even if a massive, accidental dose of radiation is received, of the order of 4000 or 5000 mSv, there is an even chance that the exposed individual will recover from the immediate threat, and, if he is to die from a radiation-induced cancer, his death is likely to be delayed by at least a decade.

In fact, the phenomenon is not restricted to the nuclear industry, but is observed with toxic materials in other industries, as evidenced by the delayed mortality from asbestosis amongst asbestos workers and miner's black lung (pneumoconiosis) amongst coal miners, for example.

Death delayed by many years is a phenomenon not addressed by the VPF at all. The VPF approach would characterise death at the age of 70, after perhaps 2 years of poor health, as a result of an accident that happened 30 years earlier, in exactly the same way as an accident that cut that person's life off immediately, at the age of 40. Lord Marshall's insight (Marshall *et al*., 1983) was that life expectancy was a much more exact and representative measure for the harm caused, particularly when that harm may be delayed by half a lifetime. The measure has the advantage that it can be applied to both immediate and delayed deaths, and put them all on the same scale, in a way that the VPF cannot do. Thus the J-value allows an objective and rational comparison amongst the hazards of the nuclear industry, those of the rail industry, the chemical industry, the health service, and so on.

A further, related point is that by completing his report at the end of 2009, Mr Spackman was able to consider only the pre-publication versions of the last 4 J-value papers published in *Process Safety and Environmental Protection*. These pre-publication papers had not yet had the benefit of the referees' comments, which led to useful changes, including the provision of examples. In particular, the requirement to split the paper, "The limits to risk aversion" (Thomas, Jones and Boyle, 2009.2) into two papers (Thomas, Jones and Boyle, 2010a,b) meant that extensive rewriting was inevitable. The availability of the clearer, published papers might have allowed Mr Spackman to avoid some of the misperceptions evident in his review.

Moreover, Mr Spackman demonstrates an imperfect knowledge and understanding of some economic and mathematical issues. Points where clarification is needed will be addressed after the next Section, which lists the nuclear J-value papers to which Mr Spackman omitted reference in his literature review. Section 3 will discuss Mr Spackman's view of utility, while Section 4 will treat the interesting question of risk aversion. Section 5 will consider the recent attempt of the NERA consortium consisting of Michael Jones-Lee, Graham Loomes and Michael Spackman to provide "a new model relating the VOLY [value of a life year] the VPF and life expectancy". The general similarities this model shares with the J-value, and its specific deficiencies will be discussed. Section 6 will consider the transparency of methods explained in the scientific literature. Section 7 will consider innovation in safety and risk management, while Section 8 will present conclusions.

The chance for scholarly debate on the issues raised by the J-value has been open since the publication of the first J-value papers in 2006 in *Process Safety and Environmental Protection* by writing to the editor of the host journal. It is conventional for the authors of a paper to be given the chance to reply, of course, with both the comments and the response subject to the normal, peer-review process.

Mr Spackman's review of the J-value literature, now that it has been published openly on the web, allows such a debate to start on the important issues and benefits associated with the J-value framework. This is wholly to be welcomed. It is to be hoped that the discussion of issues that now follows will be of general interest and benefit.

2. Nuclear J-value papers not referenced in Mr Spackman's review

The nuclear papers not considered in the review are listed in date order.

1. Thomas, P. J., Stupples, D. W., and Alghaffar, M. A., 2006c, "The life extension achieved by eliminating a prolonged radiation exposure", *Process Safety and Environmental Protection*, September, 84(B5), 16 – 27.
2. Thomas, P. J., Stupples, D. W., and Jones, R. D., 2007, "Analytical techniques for faster calculation of the life extension achieved by eliminating a prolonged radiation exposure", Trans IChemE, Part B, *Process Safety and Environmental Protection*, Vol. 85 (B3), 1 – 12.
3. Jones, R. D., Thomas, P. J., and Stupples, D. W., 2007a, "Numerical techniques for speeding up the calculation of the life extension brought about by removing a prolonged radiation exposure", *Process Safety and Environmental Protection*, July, 85(B2), 269 – 276.
4. Jones, R. D., Thomas, P. J., and Stupples, D. W., 2007b, "Erratum: Numerical techniques for speeding up the calculation of the life extension brought about by removing a prolonged radiation exposure", *Process Safety and Environmental Protection*, November, 85(B6), 599.
5. Jones, R. D. and Thomas, P. J., 2009a, "Calculating the life extension achieved by reducing nuclear accident frequency", *Process Safety and Environmental Protection*, Vol. 87, No. 2., 81 – 86, March.
6. Thomas, P. J. and Jones, R. D., 2009b, "Calculating the benefit to workers of averting a radiation exposure lasting longer than the working lifetime", *Process Safety and Environmental Protection*, Vol. 87, No. 3, 161 – 174, May.
7. Thomas, P. J. and Jones, R. D., 2009c, "The effect of the exposure time on the value of a manSievert averted", *Process Safety and Environmental Protection*, Vol. 87, No. 4, 227 – 231, July.
8. Thomas, P. J. and Jones, R. D., 2009d, "Incorporating the 2007 recommendations of the International Commission on Radiation Protection into the J-value analysis of nuclear safety systems", *Process Safety and Environmental Protection*, Vol. 87, No. 4, 245 – 253, July.

By overlooking the nuclear J-value papers, Mr Spackman gives little or no weight to one of the fundamental weaknesses of the VPF concept, namely that it cannot differentiate between the loss of amenity to the individual of his life ending immediately and the loss of amenity of his life ending in 40 years time, after decades of normal life. The fact that Mr Spackman made no reference to these papers means that his review of the J-value literature can be only partial.

3. The view of utility espoused by Mr Spackman in his review

Mr Spackman seems to be displaying a view of utility that is inconsistent with mainstream thought. For example, he has difficulty with the concept of utility as applied to large bodies such as corporations or government departments. Moreover, he reveals an unfamiliarity with the Cobb-Douglas utility function, and seems not to have understood the scaling factor applied to expected utility differences to produce the reluctance to invest. This section will address the points that Mr Spackman sees as difficulties concerning utility and utility functions.

3.1 The bodies to which utility theory applies

Mr Spackman's views on the application of utility functions are set out in his section 6.3:

> "There is no good reason to ascribe a personal [*sic*] utility function to a government department."

While it is an obvious truism that a *personal* utility function will not apply to a government department, some of the behaviour of that department can certainly be characterised using a utility function. The point was clarified over 50 years ago in the book, "Games and Decisions", by Luce and Raiffa (1957, Dover reprint 1989) , regarded by many as a classic. In Chapter 2, "Utility Theory", they state:

> "Any decision maker – a single human being or an organization – which can be thought of as having a unitary interest motivating its decisions can be treated as an individual in the theory."

Clearly government departments fall within this category of decision maker, as do commercial organisations. See also, for example, Encarnacion (1964), Sandmo (1971) and Leland (1972).

Moreover, it is standard practice in the insurance industry to apply utility functions to both the insurer (almost always a company) and the insured (which can be a single person or a partnership or a company or a state-owned body). Thus, for example, Kaas, Goovaerts, Dhaene and Denuit in the Preface to their book, "Modern Actuarial Risk Theory" (2001) state in their 2003 Preface:

> "The very existence of insurers can be explained by way of the expected utility model."

Meanwhile, that doyen of actuarial science, Professor Hans U. Gerber (University of Lausanne) explains in his textbook, Life Insurance Mathematics:

"To this end premiums are determined by a *utility function* $u(.)$; this is a function satisfying $u'(x) > 0$ and $u''(x) < 0$, and measuring the utility that the insurer has of a monetary amount x."

He then goes on to demonstrate how an insurer will find that the annual premium, when the sum insured is 100,000 units, is 1.04 times the net premium, but if the sum insured is 5,000,000 units, then the annual premium is 12.48 times the (50 times bigger) net premium. A parallel result is found in the Limits to Risk Aversion Part 2 paper (Thomas, Jones, and Boyle, 2010b), Example 2, where a £10 bn company facing a £0.95 bn loss with a probability of 0.01 would be prepared to pay up to 1.04 times the expected loss (equivalent to the net premium) but 3.8 times the expected loss when the loss was £9.5 bn if it happened, with probability 0.001. See also Gerber and Pafumi (1998).

It may be remarked, in passing, that the political scientist, Prof. Bruce Bueno de Mesquita, claims good results for his application of rational game theory, based on a simple utility function, to complex choices made by organisations including governments (Bueno de Mesquita, 2009).

Thus, when Mr Spackman seeks to restrict the region of applicability of utility functions to individuals only, he is going against established economic, actuarial and political precedent.

3.2 The Life Quality Index (LQI) as a utility function

Although he concedes later that his objection might be "a matter of wording rather than technical substance", Mr Spackman suggests in his Section 3.2.2 that

"The use of the term 'utility' in the context of the LQI is not quite right."

This may be contrasted with the description Professor Per-Olov Johansson gives of the Cobb-Douglas utility function in his widely read textbook on welfare economics (Johansson, 1991):

"Throughout this book extensive use is made of a simple analytical utility function the Cobb-Douglas utility function. Assuming two commodities, this function can be written as:

$$U = F(\mathbf{x}) = x_1^{\alpha} x_2^{1-\alpha} \qquad \text{(A.1)}$$

where x_1 and x_2 are two different commodities, $\mathbf{x}$ is a vector i.e. $\mathbf{x} = (x_1, x_2)$, α is a positive parameter $(0 < \alpha < 1)$, and U is the level of satisfaction attained by the consumer."

Per-Olov Johansson is Emeritus Professor of Economics at the Stockholm School of Economics, and a specialist in the fields of health economics (evaluation of health risks, insurance), environmental economics and industrial organisation. As an illustration of the widely known nature of the Cobb-Douglas utility function, a demonstration of its properties is given on the Wolfram website (Wolfram, 2011).

The transposition from equation (A.1) above to the Life Quality Index is given in Thomas, Stupples and Alghaffar (2006.1), one of the papers that earned Mr Spackman's praise as "superbly clear".

The fact that Mr Spackman believes that "the use of the term 'utility' in the context of the LQI is not quite right" would seem to put him at odds with mainstream economic thinking.

3.3 The scaling factor applied to the utility difference.

Mr Spackman is worried that the factor used to scale the reluctance to invest is the inverse of an absolute utility, which has "no useful meaning". He then extrapolates to the suggestion in Section 6.3 that:

> "Thus the derived concepts of 'reluctance to invest', 'permission point' and 'point of indiscriminate decision', while demonstrating impressive energy, constructive imagination and algebraic dexterity, have no practical meaning."

Mr Spackman's conclusion would seem to follow from a misunderstanding on his part. In contradiction to Mr Spackman's perception, we do, in fact, adhere to the von Neumann and Morgenstern condition for utility functions being unique to a positive linear transformation, so that only a difference in utility has validity. The relevant wording from Thomas, Jones and Boyle, 2010a is:

> "Let us compare the expected utility difference, $D(u_1, u_2|\varepsilon)$ at a given value of risk-aversion, ε, with the utility difference, $u_0(\varepsilon)$. $u_0(\varepsilon)$ is given by equation (6) and may be interpreted as the utility of the starting assets, *A*, measured relative to the utility of either one unit of money (Atkinson Utility) or of no money (Power Utility). The ratio of these two utility differences will be given by:
>
> $$R_{120}(\varepsilon) = \frac{D(u_1, u_2|\varepsilon)}{u_0(\varepsilon)} \qquad (56)$$
>
> where the subscripts in $R_{120}(\varepsilon)$ follow the subscripts of *u*. Since we may assume that the same scaling factor will apply to the numerator and denominator in any positive linear transformation corresponding to the von Neumann and Morgenstern conditions, cancellation of the common factor will mean that equation (56) is valid for any affine transformation of the utility function.
>
> $u_0(\varepsilon)$ will be strictly positive provided the starting assets, *A*, are either greater than zero, for the Power Utility function or exceed one unit for the Atkinson Utility function."

The point was explained carefully at the invited lecture given to the RAEng/InstMC/

Hazards Forum Briefing Seminar: *The limits to risk aversion: new results from safety analysis* (Thomas, 2009). A full explanation is given in Appendix A to this response.

To be fully clear: Mr Spackman is mathematically incorrect on this issue.

3.4 The unit of currency to be used when calculating the utility of a monetary asset

The last but one paragraph of Section C.2.5.2 of the review says:

> "However if the numeraire of A is changed from £1 to say £1 million, as in rows 5 to 9, there are no such dramatic changes in columns (3) or (6). Columns (6) to (8), with “absolute utility” as a denominator, have no useful meaning whatever the numeraire. But rows 5 to 9 illustrate how the so called “point of indiscriminate decision”, previously at some value of ε a little over 1, fades away when the numerical values A are scaled down by the use of a larger monetary numeraire."

The point made in the second sentence is incorrect, as has just been shown, in Section 3.3 and Appendix A.

Turning to Mr Spackman's concern with the point of indiscriminate decision, it may be observed that, while the reluctance to invest is a dimensionless variable, its value will vary slightly if the monetary value of the assets is expressed in different currencies. The point is covered in Thomas, Boyle and Kearns (2010), where it is shown that the unit of currency used (or *numéraire*) should be the "quantum of wealth", that is to say, the smallest quantum of money giving rise to a perceptible change in utility for the average adult. Evidence is adduced for the quantum of wealth being about £1 in the UK. The permission point will not "fade away" when a consistent currency unit is used, founded in people's economic experience, for the assets of an organisation and of an individual. Ideally, the same unit of currency should be used in assessing all decisions, but it is shown that little difference will occur if dollars are used instead of pounds for decisions taken by a large organisation.

In summary, the point has been covered fully in Thomas, Boyle and Kearns (2010). The use of a consistent unit of currency, founded in people's economic experience, means that no problem will occur.

4. Risk-aversion, ε, and its value

Risk-aversion was introduced as a mathematical parameter by John W. Pratt in his paper of 1964, "Risk aversion in the small and in the large" (Pratt, 1964). The value of risk-aversion, ε, has very significant implications not only for safety and risk but also for economics and, indeed, politics.

Pratt derived a measure of aversion to insurance risks that he termed the "local proportional risk aversion", now often referred to by economists as the "coefficient of relative risk aversion", and shortened to "risk-aversion" in Thomas, Jones and Boyle (2010a,b) simply to make it less cumbersome to discuss. Kenneth J. Arrow came to conclusions similar to those of Pratt in contemporaneous research on investment risk

(Arrow, 1965), except that Arrow's coefficients are the negative of Pratt's, and therefore precisely equal to the elasticity of marginal utility of income or wealth.

Thus the Pratt-Arrow work of the mid-1960's linked insurance risk and investment risk to welfare economics, as studied by John Hicks (Hicks, 1939) and Nicholas Kaldor (Kaldor, 1939) from the 1930's onwards. A further link came in 1970 with the work of Tony Atkinson (Atkinson, 1970), who showed that the same parameter, ε, could be regarded as a measure of inequality aversion as regards individuals' incomes.

The link to environmental protection comes from the parallel between taking out insurance to cover the cost of environmental damages and investing in an engineered protection system to prevent those damages happening. A similar consideration links human safety to insurance risk, and the work of Pandey, Nathwani and Lind (2003, 2006) on the Life Quality Index, as well as the J-value papers, provide the formal link between human safety and risk-aversion, ε. The Harvard economics professor, Martin Weitzman attempts to link risk-aversion and a postulated large-scale starvation event that might occur if climate change turned out to be very dramatic (Weitzman, 2009). His work has been sufficiently influential as to be referenced by Nicholas Stern when setting down his personal recommendations for managing climate change (Stern, 2009).

To summarise, risk-aversion turns out to be a key parameter in analysing:

- insurance risk
- investment risk
- safety risk
- income and wealth inequality, and hence taxation policy (in a way to be discussed shortly), and now, also,
- climate change and how to manage it.

In fact, the risk-aversion parameter provides a natural grouping of attitudes within the areas cited, so that a higher value of risk-aversion would be associated with:

- wanting more protection against random mishaps and so being willing to pay more to insure against them
- being less entrepreneurial in investment
- being less willing to take chances with safety
- wanting a more redistributive tax regime
- being more willing to pay more now to manage climate change in the future.

Although it is accepted by many that risk-seeking behaviour does occur on occasion, on the part of some people and some organisations, there is general agreement amongst economists and actuaries that most people and organisations are, most of the time, risk-averse or at least risk-neutral, implying that risk-aversion may be taken normally to be non-negative. This is consistent with a non-convex utility function for money. So there is fair agreement that $\varepsilon \geq 0$ when most people are taking decisions.

The next question is whether risk-aversion can be regarded as a constant. Two factors have combined in influencing many economists to accept that risk-aversion should be regarded as constant, at least as a pragmatic way of proceeding: (i) the fact that utility may be modelled reasonably simply when a constant risk-aversion is assumed and (ii) the difficulty of collecting data on which to build a more complex model. In fact there are fundamental reasons for taking risk-aversion as constant during each comparison between a pair of alternative outcomes, following the argument made by Thomas (2010). Risk-aversion can be expected to grow, however, as the potential loss increases as a fraction of wealth, both for the individual and for the organisation (Thomas, Jones and Boyle, 2010a, b). The insurance result from Gerber (1997), quoted in Section 3.1 of this article, points up a similar phenomenon.

Given the rather political nature of some of the bullet-points listed above, it is perhaps not too surprising that there is some dispute over what the value of risk-aversion should be. As an example where a political dimension crops up, let us consider some points made by Tony Atkinson in his 1970 paper:

> "If we look first at the absolute value of the measure, then in the United States, for example, the measure of inequality (*I*) for $\varepsilon = 1.5$ is 0.34. In other words, if incomes were equally distributed, the same level of social welfare could be achieved with only two thirds of the present national income – which is a rather striking figure. For most other countries, the figure is even lower and in the case of Mexico it is only one-half. These figures relate to one particular value of ε and it is clearly important to examine their sensitivity to changes in ε in the case of the United States. The range of variation is considerable, but *I* is less sensitive than one might at first have expected; for example, if we could agree that ε should be between 1.5 and 2.0, then *I* would lie between 0.42 and 0.34. It is also interesting to note that the potential gains from redistribution are considerable over most of the range: for $\varepsilon > 0.2$ they are greater than 5% of national income and for $\varepsilon > 0.8$ they are greater than 20%."

Rather than attempt to replicate Figure 6 of Atkinson (1970), Figure 1 to this document provides the corresponding data for the UK in 2008, which reproduce approximately the earlier American values. Clearly Inequality, *I*, is approximately linear in risk-aversion, ε. Thus the inequality values corresponding to $\varepsilon = 0.2, 0.8, 1.5$ and 2.0 are *I* = 5%, 21%, 38% and 50% respectively when UK data for 2008 are used.

Hence, following Atkinson's analysis and taking his "striking" figure corresponding to $\varepsilon = 1.5$ for the UK in 2008, 38% of national income might be regarded as a legitimate target for redistribution without any risk of reduced social welfare. On the other hand, if the figure is that recommended by the Treasury, namely $\varepsilon = 1.0$, then the target income for redistribution reduces by a third to 27%. Bring ε down to 0.83, as recommended by Pearce and Ulph (1995, 1999) and, as an approximate average, by Thomas, Jones and Kearns (2010) and by Thomas, Jones and Boyle (2010b), then the targetable income reduces to 22%. This is still substantial, but 40% down on the original amount of national income that appeared to be available for redistribution when $\varepsilon = 1.5$.

We know from Mr Spackman that at one time the Treasury's preferred value for risk-aversion was $\varepsilon = 1.5$ (Note 22 of the review, page 26), although apparently redistributional arguments such as those above were not part of the Treasury's logic at that time (same note). However, the Treasury changed from $\varepsilon = 1.5$ to $\varepsilon = 1.0$ in 2002 (Spackman, 2002) and it has maintained that guidance since (H. M. Treasury, 2003, 2011).

Mr Spackman makes the case in his extensively researched Appendix B in the review for "a value of 1.5, or perhaps a little higher", which "might be the most easily defensible". However, Mr Spackman's belief conflicts with recent economic evidence put forward not only by those working on the LQI and the J-value (Pandey, Nathwani and Lind (2006), Thomas, Jones and Kearns (2010) and Thomas, Jones and Boyle (2010b)) but also by respected economists such as Pearce and Ulph (1995, 1999), who recommend a value of 0.83. For while Mr Spackman urges us that their work "should not ... be seen as [a] very authoritative paper", it is nevertheless quoted on the Treasury website. As is the paper, Cowell and Gardner (1999), in which it is stated that:

> "experiments by their very nature may pick up on information that is special to the experiment or the respondent group.
>
> By contrast the estimates for relative risk aversion by the indirect route of inference from the intertemporal substitution elasticity are fairly consistent. Most imply values for the elasticity of marginal utility [= risk-aversion] of just below or just above one."

and

> "the low values of the elasticity [= risk-aversion] that emerge from these studies and the high values that emerge from the experimental work bracket, but do not approximate, the value of 1.5.
> ... the status of the 1.5 value is left unclear."

Meanwhile Lord Layard (Layard, Mayraz and Nickell, 2007) comes out in favour of 1.26 based on a happiness survey, a value of risk-aversion that might reasonably be regarded as "just above one", in line with Cowell and Gardiner.

It is clear that Mr Spackman has preserved his intellectual allegiance to the figure of $\varepsilon = 1.5$ that the Government has discarded. By contrast, the J-value figures of between 0.82 and 0.85 are within 20% of the Government's current view, and fully in line with the Treasury's guidance that the figure for risk-aversion is most likely to be just above or just below unity.

5. The attempt of the NERA consortium (Jones-Lee, Loomes and Spackman) to link the value of a life year (VOLY) to the value of a temporarily prevented fatality (VPF)

From the description of the J-value, in Section 6.4 of the review, as

> "a bold but very rough and ready method of estimation that cannot sensibly rival the stated preference methods now used"

one might be tempted to conclude that Mr Spackman was suggesting that the approach to valuing human lifetime using the 'one-size fits all' VPF was all that was needed. Moreover, the statement in Section 3.2.2 of the review that

> "The Canadian Life Quality Index is an ingenious concept, but it presents problems as a basis for deriving people's willingness to pay for small reductions in fatality risks. Its main limitations are threefold:
>
> 1) It assumes that the value of a prevented fatality can be satisfactorily measured by discounting the value of the expected lifetime welfare that a person would enjoy if they lived to a normal lifespan. As a principle this is very questionable"

would suggest that Mr Spackman believed that there were deep philosophical problems associated valuing human life in terms of life to come. But this would not appear to be the case. The NERA Project Team of Professor Jones-Lee, Professor Loomes and Mr Spackman, with Mr Spackman the sole in-house member, had been recently working on an approach incorporating just such a philosophy. They had put forward in the year before Mr Spackman's review:

> "a mathematical structure, which is an advance on any other of which we are aware, and which we recommend as suitable, in the present state of knowledge, for practical application" (Jones-Lee, Loomes and Spackman, 2007, 2008).

The contents of this report will presumably be well known within HSE, since it was commissioned by that body and it is published on the HSE website. In it the NERA consortium seek to overcome the deficiencies of the VPF approach with "A New Model Relating the VOLY the VPF and Life Expectancy", saying:

> "Suppose we denote by D the amount of remaining life expectancy measured in days, and take one day to be the remaining life expectancy below which small extensions (of less than a day) are zero-valued. Then we may consider a very simple functional form which has the properties we require, namely:
>
> $$\text{VPF} = \alpha D^{\beta} \text{ with } D \geq 1, \alpha > 0 \text{ and } 0 < \beta < 1$$
>
> In general terms, there is some appeal in treating the VPF as reflecting literally the "value of remaining life", or, more specifically, the sum of the value of increments in life expectancy, where these increments in total add up to remaining life expectancy itself." (*ibid.*)

This explicit recognition by Mr Spackman's NERA Project Team of using life expectancy rather than "life", irrespective of age, as the parameter that should be valued is, of course, precisely in line with the philosophy of the J-value. Indeed, the NERA equation above has some similarity to the Life Quality Index, Q, (Pandey, Nathwani and Lind, 2003, 2006, Thomas, Stupples and Alghaffar 2006.1 *et seq.*) which may be written as

$$Q = G^{q} X$$

where *X* the amount of remaining life expectancy measured in years, *G* is the annual income and *q* is the complement of the risk-aversion: $q = 1 - \varepsilon$.

Many other researchers have, of course, been aware of the problems of the VPF. A good discussion is given, for example, in the study by Rabl, Nathwani, Pandey and Hurley (2007), funded by the UK Department of Health and the European Commission as part of the ExternE series of projects. They comment that:

> "The estimation of VSL [value of a statistical life, an alternative acronym for VPF] has been a challenging and controversial topic in risk analysis."

and conclude that:

> "One of the main sources of uncertainty lies in the monetary valuation of air pollution mortality. The widely used 'value of statistical life' is based on accidents and not appropriate, and the available estimates of the value of a life-year lost due to air pollution are still very uncertain."

This leads them to the recommendation:

> "To circumvent the uncertainties of the valuation of air pollution mortality, the Life Quality Index (LQI) is proposed as an innovative policy tool because it allows integration of the key issues (discounting of life years, competing mortality risks, intertemporal trade-offs, age-dependent risks, and willingness to pay) in a consistent and transparent manner to support a credible analysis."

Meanwhile, the authors of the roughly contemporaneous NERA report commended to the HSE, and no doubt the nuclear industry, their model that costed the value of preventing a fatality based on the estimated value of the years of future expectancy. The NERA valuation model produces results that are generically similar to those produced by the J-value, especially the monotonic fall in the VPF with age.

Interestingly, although Mr Spackman describes the J-value as "very rough and ready", his own team's preferred replacement for the VPF concept contains an obvious inconsistency, in that they attempt to marry an inherently nonlinear model with linear data. The attempt to get around this problem leads them to dispense with their linear constraint at a point they admit is "arbitrary", namely when the remaining life expectancy falls below 10 years. This causes most of the curvature of their VOLY function to occur when there are less than 10 years of life expectancy remaining, with the VOLY moving rapidly upwards as the expected delay before death approaches zero. The NERA valuation model adopts the ad-hoc assumption that the model will apply only when the person is expected to last the day out, but even so the NERA value of a life year reaches almost £200,000 before the model is regarded as invalid.

Meanwhile the J-value produces a curve of the VOLY against remaining life expectancy (or expected delay before death) that is smooth and regular over the range 0 to 80 years, without resort to arbitrary and *ad hoc* measures. See Figure 2 for a comparison.

The J-value builds upon extensive work on the LQI-index by Professors Nathwani, Pandey and Lind of the University of Waterloo, as well as by Professor Dr-Ing Rackwitz of the Technical University of Munich. Moreover, 14 peer-reviewed papers, totalling some 160 pages of closely argued logic and mathematics, have been published in *Process Safety and Environmental Protection* alone by the team based at City University. Hence the epithet, "rough and ready", which Mr Spackman applies to the J-value method, would seem to be inappropriate.

By contrast, while the NERA valuation model has been documented in the 14 pages of Section 3 of the report, Jones-Lee, Loomes and Spackman (2007, 2008), it has not, to our knowledge, been published in the peer-reviewed literature. But this is not to say that the NERA valuation model is without merit. On the contrary, it demonstrates the implication of the assumption that a person will place more value on adding to his life expectancy when his future time is limited than when he is expecting to live a long time. This corresponds to the economic finding that scarce goods tend to be valued more highly. Bearing in mind that the maximum restoration of life is bounded by a life expectancy that falls with age, this assumption on its own is sufficient, when encoded into a simple mathematical model, to show that the value of removing a threat to life reduces as people get older. The "VPF" is thus a decreasing function of age. The NERA valuation model thus fulfils the useful function of providing a degree of diverse validation for the more complete J-value model.

However, Mr Spackman does appear to be displaying an odd dichotomy here. It is difficult to reconcile Mr Spackman's advocacy on one hand of his own organisation's rather simple model, which is based on valuing human life as

> "the sum of the value of *increments* in life expectancy, where these increments in total add up to remaining life expectancy itself " (Jones-Lee, Loomes and Spackman, 2007, 2008)

and recommended as

> "suitable, in the present state of knowledge, for practical application" (*ibid.*)

with his apparent lack of enthusiasm for the more sophisticated J-value method that carries out the task identified for the NERA valuation model, while avoiding its difficulties and *ad hoc* assumptions.

6. Transparency

Mr Spackman's review seems in Section 3.2.3 to be laying the charge of obscurity against the J-value:

> "In some politically sensitive contexts obscurity may be seen as a virtue."

This sentence comes immediately after the paragraph:

> "Any mathematical process is “transparent”, in the sense that a technically competent reviewer can dissect it and identify the inputs and how they are processed. However the J-value algebra is not readily transparent to users.

> Moreover the inputs incorporate many implicit assumptions about social behaviour and preferences that are not revealed. The current stated preference conventions, for all their weaknesses are much more transparent to a critical reviewer."

Thus the "obscurity" charge is in truth a complaint that the mathematics behind the J-value are hard: "the J-value algebra is not readily transparent to users". But such a charge could be levelled against any design calculations allowing the development of modern technology. But this is hardly a serious argument against the use of new, technologically advanced appliances. Just about any piece of modern equipment in everyday use, from a motor car to a television set to a mobile telephone, would fall into this category: their design equations would surely defeat most people. And it can hardly be argued that other professionals in the field of risk would be unable to understand the J-value when a large number of papers on the closely related LQI concept have been published by Lind, Nathwani, Pandey, Rackwitz and Rabl, for example.

The suggestion about unrevealed implicit assumptions is difficult to understand, not helped by the fact that Mr Spackman does not elaborate on what they are. A distinguishing feature of the J-value framework is that the assumptions have been laid out clearly in the large number of peer-reviewed, supporting papers. As noted above, 160 closely worded pages have been carried by *Process Safety and Environmental Protection* alone. Nothing is implicit in the J-value approach – the assumptions are clear and all the data are explicit and testable economic and actuarial figures.

The word "obscure" can also carry the connotation of being hard for the layman to understand. But it is the very simplicity of the J-value assessment and its non-reliance on emotion and difficult-to-justify values of VPF that make it transparent to the non-specialist. For example, an early, very readable account was carried by *The Times* (Henderson, 2005). Evidence for the ease of knowledge transfer across disciplines comes from the application of the J-value to the legal aspects of the World Trade Organization's health regulations in a paper by the lawyer, David Collins (2009).

7. Innovation

Mr Spackman's review is liberally strewn with comments, footnotes, criticisms and praise, but one recurring theme is his apparent satisfaction with the *status quo*. Thus Section 3.2.3 of the review says that:

> " the claim that this particular ratio [the J-value] has an ability to translate a variety of cost-benefit formats into a common yardstick is misleading. All that is needed to provide a common yardstick for most comparisons of the kind to which the J-value is applied is either a standard value of a prevented fatality (VPF), or, even more simply, the comparison of conventions or applications in terms of their own “cost per prevented fatality”."

Such a stance may, of course, be maintained only by ignoring the issue of valuing delayed health effects, an issue of cardinal importance to the nuclear industry, including the MOD in respect of its nuclear submarine fleet. The issue is also of great importance for other industrial diseases involving long periods between initial exposure and death, such as asbestosis and silicosis, and in controlling human harm from air pollution. By neglecting all the specifically nuclear J-value papers, the review manages to avoid addressing a concern of great industrial and regulatory importance.

Meanwhile Section 3.2.3 states:

> "it is hard to see advantages in it [the J-value] as an analytical instrument, relative to those of current conventions."

The last paragraph of Section 5.2.2 starts promisingly:

> "The suggestion near the end of Thomas, Jones and Boyle (2009) that individuals adjust their risk aversion to the point at which they are most satisfied with their choice is imaginative and has a flavour of insight into the ways that people self-rationalise."

but after an appeal to the "now well established consistent inconsistencies" seen by Mr Spackman as characterising personal choice, concludes with the flat assertion:

> "The constant-elasticity utility framework has little to contribute to this field."

And, of course, he describes the J-value finally, in Section 6.4, as

> "a bold but very rough and ready method of estimation that cannot sensibly rival the stated preference methods now used"

Despite the implicit acknowledgement of the deficiencies of the existing approaches inherent in the need to produce the NERA valuation model based on life expectancy (Jones-Lee, Loomes and Spackman, 2007, 2008), it will be seen from the above that Mr Spackman comes out in his review for "standard value", "current conventions", "well established consistent inconsistencies" and "the stated preference methods now used".

Thus the review does not seem to recognise the deficiencies in the current approaches nor the way that the J-value approach gets over these. In fact, a satisfaction with existing methods of analysis emerges that sounds worryingly complacent. For stagnation is rarely, if ever, a sensible option in any field of human activity, and it sits ill with the very important area of risk management, especially given the pressing tightness of energy supplies worldwide and the UK Government's hope that a fleet of new nuclear reactors will be built to replace existing stations with new, carbon-free generation.

The LQI, the J-value and the J-value framework represent an attempt at innovation in the field of health, safety and the environment. The academic rigour of the methods has been developed through submitting numerous papers to peer-reviewed journals and peer-reviewed international conferences. But although scientifically rigorous and founded on detailed mathematical analysis (some of which is, admittedly, of a high level and therefore difficult for the layman to understand), the output of the J-value is a single figure, which is crystal clear in its import. Thus it provides the potential for clear guidance on the application of the principle of "As low as reasonably practicable" (ALARP). ALARP is an excellent precept, but one where both HSE Inspectors and Duty-holders know that there can be significant difficulty on the ground. Protection against both accidents and recriminations in the event of an accident, for both Inspector and Duty-holder, requires that a tool be in place that can give an objective assessment to aid the final judgement and can confirm its rationality. The clarity of the J-value means that it has already been applied in the legal field (Collins, 2009), while the same feature has allowed its early communication to the general public (Henderson, 2005).

As observed by Rabl, Nathwani, Pandey and Hurley (*op. cit.*),

> "the benefits of a CBA [cost-benefit analysis] lie in what is made transparent, as well as in the answers its provides."

In this context, the J-value gives a transparent, objective assessment of "how safe is safe enough", based on the technical and economic data. It is accepted that regulators and others will use a wider base of judgement, including the need to consider socio-political issues, but the J-value provides a strong and central input into the decision-making process.

By contrast, Mr Spackman seems to regard the existing methods as acceptable. His argument against the J-value, namely that it is too mathematical – "the J-value algebra is not readily transparent to users", could be raised against just about any new thinking that could make a difference. The bias against innovation that emerges from the pages of his review is disturbing.

8. Conclusions

Although Mr Spackman's review has been useful in sparking debate and the further explanation contained here of the J-value and its associated economics and mathematics, it has to be said that the review displays flaws that ought to be of legitimate concern to the UK's Health and Safety Executive.

The fact that none of the many J-value papers concerned with the long-term health effects of nuclear radiation dose is considered in Mr Spackman's review seems difficult to understand in a purported survey of the J-value literature carried out in the context of the MOD's requirement for a better way of regulating the safety of its nuclear submarines. The omission enables the review to overlook the issue, vital to the nuclear industry and others, of the valuation of the adverse health effects when those effects are delayed by decades. The J-value is able to handle this issue in a fully scientific way, whereas the current VPF approach is unable to differentiate between death tomorrow and death in 40 years' time. The neglect in Mr Spackman's review of this vital aspect is particularly serious at a time when issues of energy price and fuel security are compelling the UK to consider building a large fleet of new nuclear power stations to maintain its economy and living standards.

The review's treatment of utility displays a perplexing unfamiliarity with the topic. It is clear that Mr Spackman believes that utility functions can be applied only to an individual, but in this he is going against established economic, actuarial and political precedent. He also demonstrates a surprising lack of awareness of the Cobb-Douglas utility function, a concept in extensive use in welfare economics. Meanwhile Mr Spackman's suggestion that an "absolute utility" is used in scaling the reluctance to invest is mathematically incorrect. Furthermore his concern over the unit of currency to be used in valuing monetary assets is resolved by using a consistent unit of currency, founded in people's economic experience.

Mr Spackman insists that the risk-aversion for individuals should be set at 1.5 or more. Mr Spackman's belief conflicts with recent evidence put forward not only by those working on the LQI and the J-value but also respected economists. Moreover his stance puts Mr Spackman into direct conflict with the view of the UK Treasury, which advocates a figure of 1.0.

Mr Spackman's lack of enthusiasm for the J-value sits oddly with his recent advocacy to the HSE of the NERA valuation model. For the NERA valuation model is based, like the J-value, on valuing life expectancy. While Mr Spackman describes the J-value as "very rough and ready", it generates results that are generically similar to those of the NERA valuation model, but is better documented and is free of the latter's identified flaws. It is therefore odd that Mr Spackman is able to recommend the NERA valuation model as "suitable, in the present state of knowledge, for practical application", but rejects the J-value in favour of "the stated preference methods now used".

Overall, the picture that Mr Spackman's review seems to present is that, in the theory of safety and risk management, the *status quo* based on the VPF is either entirely adequate, or at least cannot be improved. This is by no means the view of either other researchers or of those on the ground, who are charged with allocating resources against a number of conflicting priorities.

Thus many in the nuclear industry, for example, may feel that they are working to high safety standards and would like to have methods to indicate when all that is reasonable is being done and risks have been reduced to a level that is as low as reasonably practicable. Such methods need to be academically rigorous so that they will stand up to scrutiny and can be used in discussing the safety of their plant with the general public. Experience suggests that the existing methods do not fulfil all these roles. Innovation over and above current methods is needed if decision makers are to have objective and dependable guidance on safety issues through option selection, engineering design, satisfaction of the regulator and presentation to the general public.

Mr Spackman's review provides a welcome stimulus to discussion on the J-value. However, it suffers from serious omissions and contains inconsistencies, flaws, and disagreements with mainstream economic thinking. In addition, it appears to suggest a bias against innovation, which may act against the overall imperative to ensure that safety related expenditure decisions are rationally and objectively based. It is concluded that the review's multiple deficiencies rule it out as a basis for judgement on the merits or otherwise of the J-value framework.

9. References additional to those contained in Section 2 above.

Where more than one paper by the same authors has been published in the same year, and Mr Spackman has referred to it in his review, his notation scheme has been retained, e.g. Thomas, P. J., Stupples, D. W., and Alghaffar, M. A., 2006.1. Otherwise the publishing convention is used of attaching an alphabetic suffix, e.g. Thomas, P. J., Jones, R. D. and Boyle, W. J. O., 2010a.

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Appendix A. The measurement of utility relative to a low value, either 1 unit of currency or none

Von Neumann and Morgenstern (1944, 2007) were able to justify in a mathematically rigorous, axiomatic way that utility functions, $v(x)$, arising solely from evidence of preferences would take the form

$$v(x) = ku(x) + h \qquad \text{(A.1)}$$

where *h* and *k* are arbitrary constants, subject to $k > 0$ and $u(x)$ is the characteristic utility function found by putting *h* = 0 and *k* = 1. By equation (A.1), differences between utilities will yield:

$$v(x_1) - v(x_2) = k(u(x_1) - u(x_2)) \qquad \text{(A.2)}$$

From equation (A.2) the sign of $v(x_1) - v(x_2)$ will be the same as the sign of $u(x_1) - u(x_2)$, and so a decision based on the direction of the change in utility will be the same whether the characteristic utility function, $u(x)$, is used or the positive linear transformation, $v(x)$. Each utility function belonging to such a family, having its own value of *k* and *h*, will produce a cardinal scale. The most familiar examples of such cardinal scales in the physical world are the Fahrenheit and Celsius scales for temperature. These may be contrasted with the absolute scales of temperature, whether Kelvin or Rankine.

Interestingly, von Neumann did not rule out the possibility that further research and insight could produce an absolute scale for utility (see von Neumann and Morgenstern (1944, 2007), chapter 3, The notion of utility, section 3.2, Principles of measurement.). Economists have gone further, and, as a matter of pragmatism, developed a range of theories for the measurability of utility, some less restrictive than the von Neumann, cardinal scale. These vary from:

- ordinal scale, where the fewest assumptions are needed. The order of utilities matters, but the difference between utilities conveys no information
- cardinal scale as per von Neumann
- ratio scale (e.g. when *h* = 0, but see below), through to
- absolute measurability.

See Boadway and Bruce (1985), Chapter 5, "Social welfare orderings: requirements and possibilities", Section 3.2. and also Johansson (1991), Chapter 3.

In contradiction to Mr Spackman's misperception, we adhere to the von Neumann condition throughout Thomas, Jones and Boyle (2010a and 2010b), as will now be explained.

In line with equation (A.1) above, the general form of the Power utility function, $v(x)$, that allows for positive linear transformations, is

$$v(x) = ku(x) + h = kU_\varepsilon(x) + h = kx^{1-\varepsilon} + h \qquad \text{(A.3)}$$

where *h* and *k* are constants, with $k > 0$ and $u(x) = U_\varepsilon(x) = x^{1-\varepsilon}$ is the characteristic Power utility function. The utility of monetary assets, *x*, relative to the utility of no assets, is therefore the difference:

$$\begin{aligned} v(x) - v(0) &= ku(x) + h - (ku(0) + h) = kx^{1-\varepsilon} + h - k \times 0 - h \\ &= kx^{1-\varepsilon} = ku(x) \end{aligned} \tag{A.4}$$

Thus in the case of a Power utility, the utility of assets relative to the utility of no assets is a scaled version of the characteristic utility, where the scaling factor is the same as would apply to all utility differences. Thus the scaling factor, *k*, will cancel in any ratio of two utility differences. This is, of course, what happens in the formation of the reluctance to invest based on the Power utility function, R_{120P} (equation (59) of Thomas, Jones and Boyle (2010a)). It might be argued from a common-sense point of view that the utility of no money should be zero, but in fact, as demonstrated, there is no need to do so.

Scaling equation (A.3) by the convenience multiplier, $1/(1-\varepsilon)$, gives the general form for the utility of assets, *x*, as:

$$v(x) = ku(x) + h = k\frac{x^{1-\varepsilon}}{1-\varepsilon} + h \tag{A.5}$$

where, once again, *h* and *k* are any constants, subject to $k > 0$. Thus the utility of assets worth one monetary unit is

$$v(1) = k\frac{1^{1-\varepsilon}}{1-\varepsilon} + h \tag{A.6}$$

Subtracting (A.6) from (A.5) gives:

$$v(x) - v(1) = k\frac{x^{1-\varepsilon}}{1-\varepsilon} - k\frac{1^{1-\varepsilon}}{1-\varepsilon} = \frac{k}{1-\varepsilon}x^{1-\varepsilon} - \frac{k}{1-\varepsilon} = k\frac{x^{1-\varepsilon} - 1}{1-\varepsilon} = ku(x) \tag{A.7}$$

which is, of course, a scaled version of the characteristic Atkinson utility function. Thus the Atkinson function can be seen to be the utility of assets, *x*, relative to the utility of one monetary unit of assets. Differences between assets, *x* and *y*, will result in the utility difference:

$$\begin{aligned} v(x) - v(y) &= v(x) - v(1) - (v(y) - v(1)) = k\frac{x^{1-\varepsilon} - 1}{1-\varepsilon} - k\frac{y^{1-\varepsilon} - 1}{1-\varepsilon} \\ &= k(u(x) - u(y)) \end{aligned} \tag{A.8}$$

which is, of course, simply the scaled version of the difference in the characteristic Atkinson utilities. Once again, the scaling factor, *k*, will drop out when the ratio of any two utility differences is taken. Of course, this is what occurs in the formation of the reluctance to invest based on the Atkinson utility function, R_{120A} (equation (60) of Thomas, Jones and Boyle. (2010a)).

It is clear from equations (A.4) and (A.7) that the characteristic Power utility is a unit-scaled version of the utility of assets relative to datum of the utility of no assets. Meanwhile the characteristic Atkinson utility function is a unit-scaled version of the utility of assets relative to the datum of the utility of assets worth one monetary unit.

Appendix B. Biographies

<u>Philip Thomas</u>

Professor Thomas graduated in Cybernetics and Instrument Physics from Reading University with a BSc (First Class Honours) and University Prize. He then gained over 20 years' experience working in the chemical and nuclear industries. His initial posting was with ICI, where he spent 5 years devising control systems for large chemical plants, developing and making extensive use of dynamic simulation. He then joined the UKAEA/ AEA Technology, where he headed up Departments concerned with: C&I product development for nuclear and non-nuclear use, non-destructive testing technology and materials research, and research into remote handling and nuclear decommissioning, where he was Customer Project Manager for the green-field decommissioning of the 100MWth/33MWe Windscale AGR.

He was appointed a Visiting Professor at City University in 1995, and then took up the Chair in Engineering Development in 2000. His research interests are wide-ranging and he has published over 100 papers on control, instrumentation, nuclear decommissioning, risk assessment and economics, as well as 30 industrial reports. His book, *Simulation of Industrial Processes for Control Engineers*, was published in 1999 and received praise from reviewers as the new standard text for practitioners of dynamic simulation. His work in risk analysis and management has excited the interest of the national and international press; his findings have been covered by both broadsheets and the popular press, and he has given several radio and television interviews. His research is currently concentrated on the J-value framework of techniques his team has developed to assess the appropriate level of spending to avert risks to people and the environment. City University awarded him a DSc in 2005 for his contribution to science and engineering, and his work has received sponsorship from a number of organisations, including EPSRC, ESRC, Government departments, EU and industrial companies.

Professor Thomas has served on national committees of the Institute of Measurement and Control since 1979, and as President in 2001. He is currently Honorary Treasurer of that organisation. He has served on senior committees of the IEE/IET, on the Board of Trustees of the British Nuclear Energy Society, on the Education and Training Committee of the Nuclear Institute and as Chairman of the Nuclear Academia-Industry Liaison Society (NAILS), 2003 – 2006. He is Chairman of the Nuclear Technology Education Consortium Board, which oversees the provision of postgraduate education in eleven universities in the UK, and for which he teaches the Risk Management MSc module. He sits on the Editorial Board of *Process Safety and Environmental Protection*, Official Journal of the European Federation of Chemical Engineering: Part B.

The Institute of Measurement and Control awarded him its ICI Prize in 1984, 1987 and 1997, and presented him with the Honeywell International Medal 2004 for distinguished work in control. The Worshipful Company of Scientific Instrument Makers awarded him its 2006 Best Paper Prize. The IChemE presented him with "Most Cited Article" certificates for each of his first three J-value papers, awarded for the top 75 most cited articles published across the IChemE's 4 journals, 2006 – 2009.

Roger Jones

Roger Jones graduated with a first-class honours in Pure Mathematics from Swansea University, and was subsequently awarded an MSc from Birmingham University in Mathematical Statistics. After carrying out applied research at the Rothamsted Experimental Station he took up the post of Senior Lecturer in mathematics at the University of Hertfordshire. He has also taught at the postgraduate level at City University. For the past 5 years, he has been carrying out research within City University's Risk Management, Reliability and Maintenance Group, where he is Honorary Visiting Fellow.

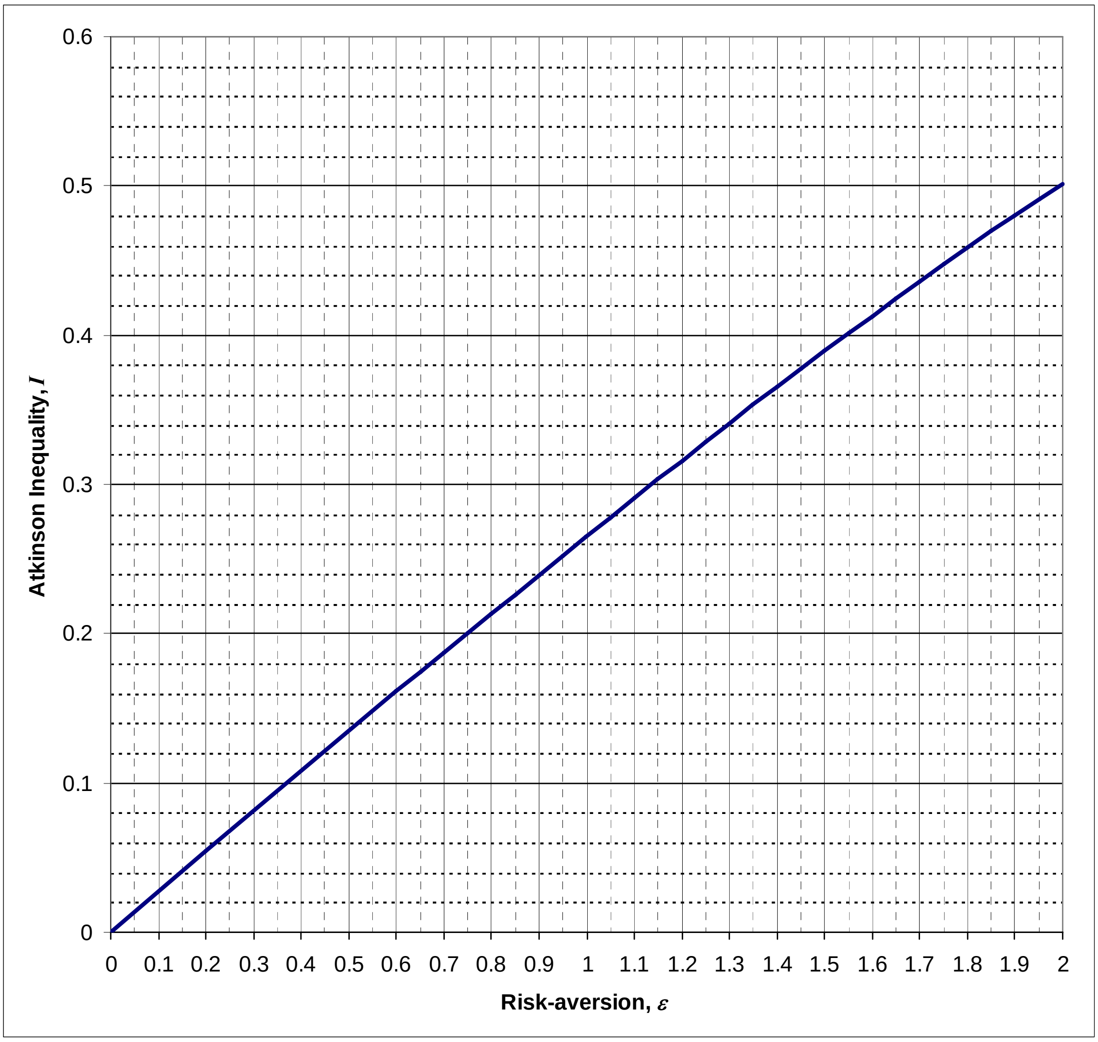

Figure 1. Atkinson's Inequality versus risk-aversion. UK data, 2008

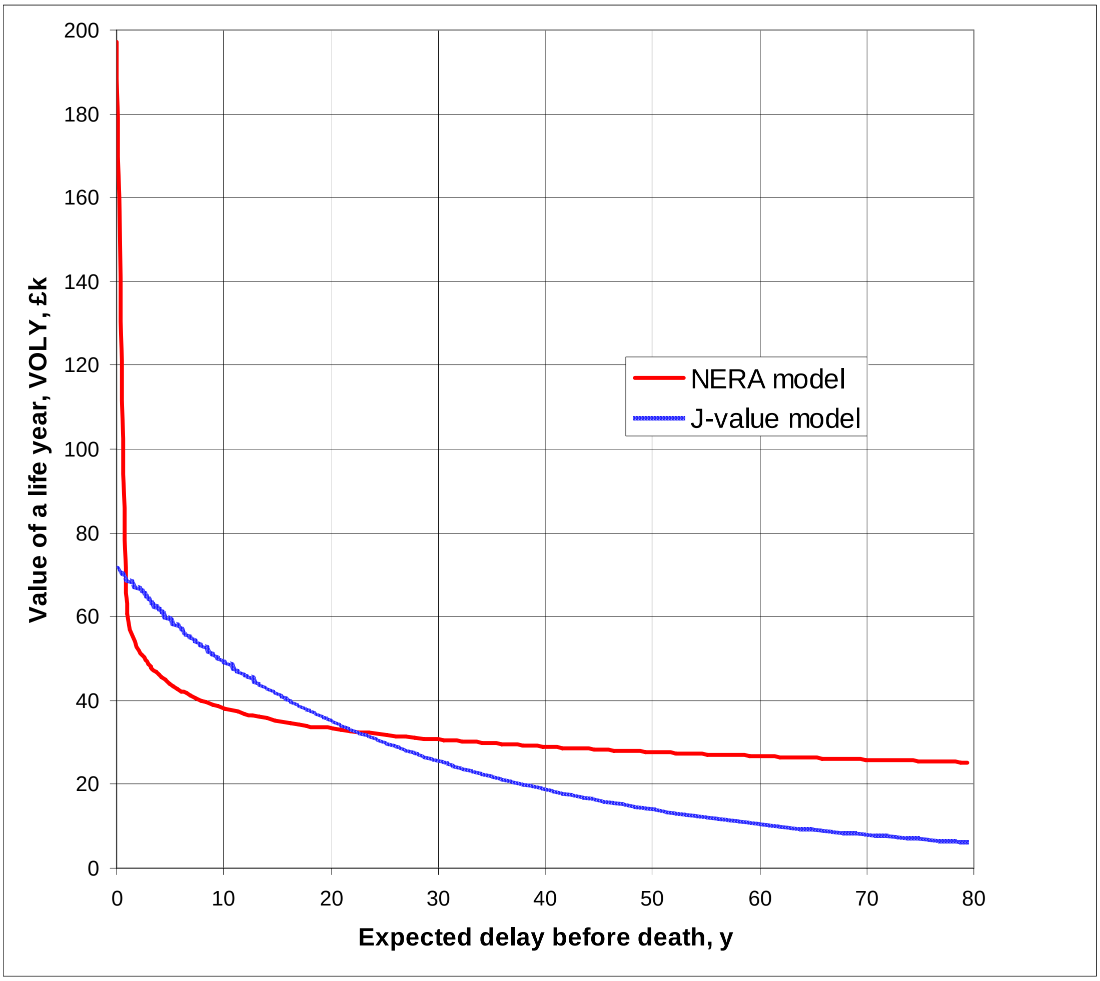

Figure 2. Value of a life year, VOLY, against the expected delay before death. Comparison between NERA valuation model and J-value model

(The J-value has been adjusted to give an average VPF of £1.4 M, the figure cited by NERA)